Hover Performance Prediction Methods

II. Blade Element Theory
Preliminary Remarks

• Momentum theory gives rapid, back-of-the-envelope estimates of Power.
• This approach is sufficient to size a rotor (i.e. select the disk area) for a given power plant (engine), and a given gross weight.
• This approach is not adequate for designing the rotor.
Drawbacks of Momentum Theory

• It does not take into account
  – Number of blades
  – Airfoil characteristics (lift, drag, angle of zero lift)
  – Blade planform (taper, sweep, root cut-out)
  – Blade twist distribution
  – Compressibility effects
Blade Element Theory

- Blade Element Theory rectifies many of these drawbacks. First proposed by Drzewiecki in 1892.
- It is a “strip” theory. The blade is divided into a number of strips, of width $\Delta r$.
- The lift generated by that strip, and the power consumed by that strip, are computed using 2-D airfoil aerodynamics.
- The contributions from all the strips from all the blades are summed up to get total thrust, and total power.
Typical Blade Section (Strip)

\[ T = b \int_{\text{Cut-Out}}^{\text{Tip}} dT \]

\[ P = b \int_{\text{Cut-Out}}^{\text{Tip}} dP \]

Root Cut-out
Typical Airfoil Section

\[ \phi = \arctan\left( \frac{V + v}{\Omega r} \right) \]

\[ \alpha_{\text{effective}} = \theta - \phi \]
Sectional Forces

Once the effective angle of attack is known, we can look-up the lift and drag coefficients for the airfoil section at that strip. We can subsequently compute sectional lift and drag forces per foot (or meter) of span.

\[
\Delta L = \frac{1}{2} \rho (U_T^2 + U_P^2) c C_l \\
\Delta D = \frac{1}{2} \rho (U_T^2 + U_P^2) c C_d
\]

\[U_T = \omega r\]
\[U_P = V + v\]

These forces will be normal to and along the total velocity vector.
Rotation of Forces

\[ dT = (\Delta L \cos(\phi) - \Delta D \sin(\phi))dr \]
\[ = \frac{1}{2} \rho (U_T^2 + U_P^2) c (C_l \cos(\phi) - C_d \sin(\phi))dr \]

\[ dF_x = (\Delta D \cos(\phi) + \Delta L \sin(\phi))dr \]
\[ = \frac{1}{2} \rho (U_T^2 + U_P^2) c (C_d \cos(\phi) + C_l \sin(\phi))dr \]

\[ dP = U_T dF_x = \Omega r dF_X \]
Approximate Expressions

• The integration (or summation of forces) can only be done numerically.

• A spreadsheet may be designed. A sample spreadsheet is being provided as part of the course notes.

• In some simple cases, analytical expressions may be obtained.
Closed Form Integrations

- The chord $c$ is constant. Simple linear twist.
- The inflow velocity $v$ and climb velocity $V$ are small. Thus, $\phi << 1$.
- We can approximate $\cos(\phi)$ by unity, and approximate $\sin(\phi)$ by $(\phi)$.
- The lift coefficient is a linear function of the effective angle of attack, that is, $C_l = a(\theta - \phi)$ where $a$ is the lift curve slope.
- For low speeds, $a$ may be set equal to 5.7 per radian.
- $C_d$ is small. So, $C_d \sin(\phi)$ may be neglected.
- The in-plane velocity $\Omega r$ is much larger than the normal component $V+v$ over most of the rotor.
Closed Form Expressions

\[ T = \frac{1}{2} \rho cb a \Omega^2 \int_{r=0}^{r=R} \left( \theta - \frac{V}{\Omega r} - \frac{V}{\Omega r} \right) r^2 dr \]

\[ P = \frac{1}{2} \rho cb \]

\[ \Omega^3 \int_{r=0}^{r=R} \left[ a \left( \theta - \frac{V}{\Omega r} - \frac{V}{\Omega r} \right) \left( \frac{V}{\Omega r} + \frac{V}{\Omega r} \right) + C_d \right] r^3 dr \]
Linearly Twisted Rotor: Thrust

Here, we assume that the pitch angle varies as

\[ \theta = E + Fr \]

\[ T = \frac{b}{2} \rho \Omega^2 ca \left[ \frac{1}{3} \left( E + \frac{3}{4} FR \right) - \frac{V + \nu}{2\Omega R} \right] R^3 = \frac{b}{2} \rho c a (\Omega R)^2 R \left[ \frac{\theta_{.75R}}{3} - \lambda / 2 \right] \]

\[ C_T = \frac{abc}{2\pi R} \left[ \frac{\theta_{.75}}{3} - \lambda / 2 \right] = \frac{a \sigma}{2} \left[ \frac{\theta_{.75R}}{3} - \lambda / 2 \right] \]

where

\[ \sigma = \text{solidity} = \frac{\text{Blade Area}}{\text{Disk Area}} = \frac{bc}{\pi R} \]

\[ a = \text{Lift Curve slope} (\sim 2\pi) \]

\[ \lambda = \text{Inflow Ratio} = \frac{V + \nu}{\Omega R} \]
Linearly Twisted Rotor

Notice that the thrust coefficient is linearly proportional to the pitch angle $\theta$ at the 75% Radius.

This is why the pitch angle is usually defined at 75% R in industry.

The expression for power may be integrated in a similar manner, if the drag coefficient $C_d$ is assumed to be a constant, equal to $C_{d0}$.

$$C_P = \lambda C_T + \frac{\sigma C_{d0}}{8}$$

Induced Power  Profile Power
Closed Form Expressions for Ideally Twisted Rotor

\[ \theta = \frac{\theta_{tip} R}{r} \]

\[ C_T = \frac{\sigma a}{4} \left( \theta_{tip} - \lambda \right) \]

\[ C_P = \lambda C_T + \frac{\sigma C_d 0}{8} \]

Same as linearly Twisted rotor!
Figure of Merit according to Blade Element Theory

\[ FM = \frac{\lambda C_T}{\lambda C_T + \sigma C_{d0} / 8} \]

where,

\[ \lambda = \text{Inflow Ratio} = (V + v)/\Omega R \]

\[ \sigma = \text{Solidity} = \text{Blade Area/Disk Area} \]

High solidity (lot of blades, wide-chord, large blade area) leads to higher Power consumption, and lower figure of merit.

Figure of merit can be improved with the use of low drag airfoils.
Average Lift Coefficient

- Let us assume that every section of the entire rotor is operating at an optimum lift coefficient.
- Let us assume the rotor is untapered.

Rotor will stall if average lift coefficient exceeds 1.2, or so.

Thus, in practice, $C_T/\sigma$ is limited to 0.2 or so.

$$\bar{C}_1 = 6 \frac{C_T}{\sigma}$$

$$T = b \int_{0}^{R} \frac{1}{2} \rho c (\Omega r)^2 \bar{C}_1 dr = \frac{\rho bc \bar{C}_1 \Omega^2 R^3}{6}$$

$$C_T = \frac{T}{\rho \pi R^2 (\Omega R)^2} = \frac{bc}{\pi R} \frac{\bar{C}_1}{6} = \sigma \frac{\bar{C}_1}{6}$$
Optimum Lift Coefficient in Hover

\[ FM = \frac{\lambda C_T}{\lambda C_T + \frac{\sigma C_{d0}}{8}} \]

In hover, \( \lambda = \sqrt{\frac{C_T}{2}} \)

\[ FM = \frac{C_T^{3/2}}{\sqrt{2}} + \frac{\sigma C_{d0}}{\sqrt{2}} \]

If \( C_T = \sigma \bar{C}_l / 6 \)

FM is maximized if \( C_{d0} / \bar{C}_l^{3/2} \)
is minimized.