Helicopter Performance

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Part 1 - Recap

• We looked at the history of helicopters.
• We studied ways of overcoming reactive torque – tail rotors, co-axial rotors, tilt-rotors, tip jets, NOTAR etc.
• We looked at a number of ways predicting helicopter performance in hover, and climb.
Momentum theory-
Induced Velocities

The excess velocity in the Far wake is twice the induced Velocity at the rotor disk.

To accommodate this excess Velocity, the stream tube has to contract.
Induced Velocity at the Rotor Disk

Now we can compute the induced velocity at the rotor disk in terms of thrust $T$.

$$T = 2\rho A v (V + v)$$

$$v = -\frac{V}{2} + \sqrt{\left(\frac{V}{2}\right)^2 + \frac{T}{2\rho A}}$$

In Hover, $V = 0$

$$v = \sqrt{\frac{T}{2\rho A}}$$
Ideal Power Consumed by the Rotor

\[ P = T \left[ \frac{V}{2} + \sqrt\left(\frac{V}{2}\right)^2 + \frac{T}{2 \rho A} \right] \]

Use this during conceptual design to size rotor, select engines

In hover, ideal power \( P = T \sqrt{\frac{T}{2 \rho A}} \)
Figure of Merit

• Figure of merit is defined as the ratio of ideal power for a rotor in hover obtained from momentum theory and the actual power consumed by the rotor.

• For most rotors, it is between 0.7 and 0.8.

\[ FM = \frac{\text{Ideal Power in Hover}}{\text{Actual Power in Hover}} \]
Non-Dimensional Forms

Thrust, Torque, and Power are usually expressed in non-dimensional form.

\[ C_T = \text{Thrust Coefficient} = \frac{T}{\rho A (\Omega R)^2} \]

\[ C_P = \text{Power Coefficient} = \frac{P}{\rho A (\Omega R)^3} \]

\[ C_Q = \text{Torque Coefficient} = \frac{Q}{\rho AR (\Omega R)^2} \]

In hover, Power = Angular velocity x Torque

\[ P = \Omega Q \]

\[ C_p = C_Q \]
Blade Element Theory

Use this during preliminary design, when
You have selected airfoils, number of blades,
Planform (taper), twist, solidity, etc.

\[ T = b \int_{Cut-Out}^{Tip} dT \]

\[ P = b \int_{Cut-Out}^{Tip} dP \]
Typical Airfoil Section

\[ \phi = \arctan \left( \frac{V}{\Omega r} \right) \]

\[ \alpha_i = \arctan \left( \frac{V}{\Omega r} \right) \]

Zero Lift Line

Effective Angle of Attack = \( \theta - \alpha_i - \phi \)
Closed Form Expressions

\[ T = \frac{1}{2} \rho \varepsilon \Omega \int_{r=0}^{r=R} \left( \theta - \frac{V}{\Omega r} - \frac{v}{\Omega r} \right) r^2 dr \]

\[ P = \frac{1}{2} \rho \varepsilon \Omega^3 \int_{r=0}^{r=R} \left[ \left( \theta - \frac{V}{\Omega r} - \frac{v}{\Omega r} \right) \left( \frac{V}{\Omega r} + \frac{v}{\Omega r} \right) + C_d \right] r^3 dr \]
Average Lift Coefficient

• Let us assume that every section of the entire rotor is operating at an optimum lift coefficient.

• Let us assume the rotor is untapered.

\[
\text{Average Lift Coefficient} = \bar{C}_1
\]

\[
T = b \int_0^R \frac{1}{2} \rho c (\Omega r)^2 \bar{C}_1 dr = \frac{\rho bc \bar{C}_1 \Omega^2 R^3}{6}
\]

\[
C_T = \frac{T}{\rho \pi R^2 (\Omega R)^2} = \frac{bc \bar{C}_1}{\pi R} = \sigma \frac{\bar{C}_1}{6}
\]

\[
\bar{C}_1 = 6 \frac{C_T}{\sigma}
\]

Rotor will stall if average lift coefficient exceeds 1.2, or so. Thus, in practice, \(C_T/\sigma\) is limited to 0.2 or so.

Use this to select solidity \(\sigma\), during design.
Combined Blade Element-Momentum Theory

Equate the Thrust for the Elements from the
Momentum and Blade Element Approaches

\[
\lambda^2 + \left( \frac{\sigma a}{8} - \lambda_c \right) \lambda - \frac{\sigma a}{8} \theta \frac{r}{R} = 0
\]

where,

\[
\lambda_c = \frac{V}{\Omega R}
\]

\[
\lambda = \frac{V + \nu}{\Omega R}
\]

\[
\lambda = \sqrt{\left( \frac{\sigma a}{16} - \frac{\lambda_c}{2} \right)^2 + \frac{\sigma a}{8} \theta \frac{r}{R} - \left( \frac{\sigma a}{16} - \frac{\lambda_c}{2} \right)}
\]

Total Inflow Velocity from Combined Blade Element-Momentum Theory
Vortex Theory

- We also looked at modeling the tip vortices from the rotor using a prescribed form, and computing induced velocity $v$ due to these vortices using Biot-Savart law.
We next looked at..

- What happens when a helicopter vertically descends, perhaps due to a loss of engine power.
- The rotor may grow through four phases:
  - Hover
  - Vortex Ring State
  - Turbulent Wake State
  - Wind Mill Brake State
- We mentioned that the rotor is as good as a parachute with drag coefficient 1.4
We finally looked at:

- Coning Angle of the blades
- Lock Number

\[ I\Omega^2 \beta_0 = \int_{r=0}^{r=R} \frac{1}{2} \rho c (\Omega r)^2 r C_l dr \]

\[ \beta_0 = \frac{\int_{r=0}^{r=R} \frac{1}{2} \rho c r^3 C_l dr}{I} = \frac{\rho a c R^4}{I} \int_{r=0}^{r=R} \left( \frac{r}{R} \right)^3 \alpha_{\text{effective}} d \frac{r}{R} \]

Lock Number, \( \gamma \)
Recap- Part 2

• We studied how to compute the induced velocity $v$ through a rotor in forward flight.

Glauert equation in non-dimensional form:

$$C_T = 2\lambda_i \sqrt{\mu^2 + \left(\mu \tan \alpha_{TPP} + \lambda_i\right)^2}$$

$$\lambda_i \approx \frac{C_T}{2\mu} \quad \text{if} \quad \mu \geq 0.2$$
We studied a simplified picture of Force Balance

\[ T \sin \alpha_{TPP} = D \]
\[ T \cos \alpha_{TPP} = W \]
Force Balance in Forward Flight

- Thrust, $T$
- Vehicle Drag, $D$
- Weight, $W$

Flight Direction
We refined the Horizontal Force Balance

Total Drag = Fuselage Drag ($D_F$) + H-force on main rotor ($H_M$) + H-force on the tail rotor ($H_T$)
We included fuselage lift in Vertical Force Balance.

Vertical Force = GW - Lift generated by the fuselage, $L_F$
We discussed how to estimate Power Consumption in Forward Flight

\[ C_P = C_T \lambda_i + \frac{1}{2} \frac{f}{A} \mu^3 + \frac{\sigma C_{d,0}}{8} (1 + 3 \mu^2) \]

*Induced Power, \( i \)*

*Parasite Power*

*Total Power*

*Blade Profile Power*
We discussed hinge and control mechanisms.
We discussed...

- Three coordinate systems
  - Shaft Plane
  - Tip path Plane
  - Control Plane

- Blade Flapping Dynamics Equation:

\[ I\ddot{\beta} + I\Omega^2 \beta = \int_{Root}^{Tip} \frac{1}{2} \rho c C_l \left( \Omega r + V_\infty \sin \psi \right)^2 r dr \]

Natural frequency = \Omega

If a first harmonic input at the resonance frequency is imposed, The blades will respond 90 degrees later!
We discussed..

• Sectional angle of attack
• This angle of attack can be achieved either by flapping or by cyclic pitch.
• One degree of pitch is equivalent to one degree of flapping as far as the blade is concerned.

\[
\alpha_{\text{effective}} = \frac{U_T \theta - U_P}{U_T} = \frac{1}{U_T} \left[ \Omega r \left\{ \theta_0 + (\theta_{1c} - \beta_{1s}) \cos \psi + (\theta_{1s} + \beta_{1c}) \sin \psi \right\} 
\right.
\]

\[
\left. + V \theta_0 \sin \psi + V (\theta_{1c} - \beta_{1s}) \cos \psi \sin \psi 
\right]
\]

\[
+ V (\theta_{1s} + \beta_{1c}) \sin^2 \psi - V \beta_0 \cos \psi
\]

\[-V \alpha_{TBP} - \nu \]
We discussed..

- How to compute the sectional loads.
- How to integrate them radially, and average them azimuthally to get total thrust, total torque, power, H-force, and Y-force.

\[
C_T = \frac{\sigma_0}{2} \left[ \frac{\theta_0}{3} \left\{ 1 + \frac{3}{2} \mu^2 \right\} + \frac{\theta_{tw}}{4} \left\{ 1 + \mu^2 \right\} + \mu \frac{\theta_{1s}}{2} - \frac{\lambda_{TPP}}{2} \right]
\]

\[
C_{P,0} = \frac{\sigma C_{d,0}}{8} \left[ 1 + 3\mu^2 \right]
\]

\[
C_{Q,0} = \frac{\sigma C_{d,0}}{8} \left[ 1 + \mu^2 \right]
\]
Azimuthal Variation of The Blade Lift

\[ \frac{L}{\rho A (\Omega R)^2} \]

\[ \Psi \text{ (deg)} \]

\[ \frac{r}{R} = 0.6 \]
\[ \frac{r}{R} = 0.8 \]
\[ \frac{r}{R} = 1.0 \]
\[ \frac{r}{R} = 0.4 \]
We discussed..

• How to trim an entire vehicle, taking into account fuselage lift and drag characteristics.

• We also discussed how to trim an entire vehicle, when it is in autorotative descent.

• Trim means all the forces and moments on the vehicle are balanced, and the power required equals power available.
Vehicle Performance Methods

An Overview
Performance Engineer needs to tell

- How high?  - Service altitude
- How fast?   - Maximum forward speed
- How far?    - Vehicle range
- How long?   - Vehicle endurance
We will assume

- Engine performance data is available from engine manufacturer.
  - This information comprises of variation of shaft horse power and fuel consumption with altitude, forward speed, and temperature at various settings (max continuous power, max take-off power, etc.)
  - Estimates of installed power losses are available.
Installed Power losses are caused by:

- Inlet duct total pressure losses due to skin friction (1 to 4%)
- Inlet total pressure losses due to particle separators (3 to 10%)
- Exhaust duct total pressure losses due to skin friction (2%), and due to infra-red suppressors (3% to 15%).
- Exhaust gas reingestion (1 to 4 degrees inlet temp rise)
- Compressor bleed (1 to 20%)
- Engine mounted accessories (addition 100 HP)
- Transmission losses
We also need vertical drag or download of the fuselage in hover. This is done experimentally, or by modeling the fuselage as a series of bluff body segments.
We also need corrections for

- Main rotor-tail rotor interference. The tail rotor is immersed in the downwash of the main rotor.
- The tail rotor will need to generate more thrust than estimated, because some of the thrust generated is lost by the interference effects.
- Empirical curves that plot this interference effect as a function of separation distance between the main and tail rotor shafts are constructed.
We also need models/corrections for:

- **Ground effect**: This reduces the inflow through the rotor, and has the beneficial effect of reducing the torque needed.
- **Fuselage plus hub Parasite Drag coefficient** (or equivalent flat plate area, $f$) as a function of fuselage angle of attack.
- **Miscellaneous drag** due to antennae, Pitot probes, hinges, latches, steps, etc.
- One of the major efforts in performance studies is an accurate fuselage drag estimate.
- The fuselage drag is expressed as equivalent flat plate area.
## Equivalent Flat Plate Areas (Ref. Prouty)

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Ground Effect in Hover

When operating near ground, induced velocity decreases. Induced power decreases. Performance increases, since total power consumed is less.

IGE: In ground effect
OGE: Out of ground effect

\[
\frac{1}{0.9926 + 0.0379 \left( \frac{D}{z} \right)^2}
\]
Power needed for vertical climb

• Sum of:
  – Rate of change of potential energy
  – Main rotor power (based on Thrust equals Gross weight plus fuselage vertical drag)
  – Tail rotor power taking into account extra thrust needed to overcome interference
  – Transmission and Installation Losses

• This is compared against the available power, to determine the maximum GW that the vehicle can lift.
• When power required equals power available under zero climb rate, absolute ceiling has been reached.
• When there is just enough power left to climb at 100 ft/min, service ceiling is reached.
• These calculations are plotted as charts.
Example

- **Gross Weight** = 16000 lb
- **Main rotor**
  - \( R=27 \text{ ft}, c=1.7 \text{ ft}, \sigma=0.082, b=4, \Omega R=725 \text{ ft/sec} \)
- **Tail rotor**
  - \( R=5.5 \text{ ft}, c=0.8 \text{ ft}, \sigma=0.19, b=4, \Omega R=685 \text{ ft/sec} \)
- **Distance between main and tail rotor** = 32.5 ft
- **Use** \( \kappa=1.15 \), **Drag coefficient** = 0.008
- **Available power less losses** = 3000 HP at sea-level less 10%
- **Neglect download.**
- **Find absolute ceiling as density altitude.**
Variation of Density with altitude

\[
\frac{\rho}{\rho_{sea\text{-level}}} = \left[ 1 - \frac{0.00198h}{288.16} \right]^{4.2553}
\]

Variation of Power with Altitude:

\[
\frac{P}{P_{sea\text{-level}}} = \frac{\rho}{\rho_{sea\text{-level}}}
\]
• Assume an altitude, h. Compute density, \( \rho \).
• Do the following for main rotor:
  – Find main rotor area A
  – Find \( v \) as \( [T/(2\rho A)]^{1/2} \) Note T = Vehicle weight in lbf.
  – Insert supplied values of \( \kappa \), \( C_{d0} \), \( W \) to find main rotor \( P \).
  – Divide this power by angular velocity \( \Omega \) to get main rotor torque.
  – Divide this by the distance between the two rotor shafts to get tail rotor thrust.
• Now that the tail rotor thrust is known, find tail rotor power in the same way as the main rotor.
• Add main rotor and tail rotor powers. Compare with available power.
• Increase altitude, until required power = available power.
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Hover Ceiling with Take-Off Power

Hover Ceiling vs. Gross Weight

- Standard Day
- 95 deg F
Power needed in Hover

Engine HP vs. Gross Weight

High Altitude

Low Altitude
Forward Flight Performance

• Power required is the sum of
  – Induced power of the main rotor, $\kappa T_v$
  – Profile power of the main rotor
  – Tail rotor induced power and profile power
  – Fuselage parasite drag times forward speed
Power Requirements

- Low altitude
- High altitude
- Intermediate altitude

Onset of stall
Severe Stall

Power vs. Forward Speed graph
Maximum Speed

- Select GW, atmospheric density, density ratio $\frac{\rho}{\rho}$ at sea-level
- Use engine charts to find total power available at sea level
- Divide power by density ratio to find excess power needed at higher altitudes.
- Match power required to (Power available at sea level + Excess Power needed at high altitudes), to get a first estimate for forward flight.
- If compressibility effects are present, correct sectional drag coefficient to include wave drag.
- Iterate on the maximum speed.
Maximum Speed

Maximum speed in ft/sec or knots

GW ~ 10,000 lb

GW ~ 24,000 lb

Continuous Power

Take-off Power

120 knots

180 knots

Maximum speed in ft/sec or knots

Take-off Power
Equivalent Lift to Drag Ratio

- It is at times of interest to compare a rotor with that of a fixed wing.
- This comparison is done in terms of L/D of the rotor.
- Fuselage effects are excluded in this comparisons.
- L is the vertical component of rotor thrust.
- D = Main rotor power/Forward Speed – Fuselage Drag
Equivalent Rotor L/D

Forward Speed

L/D

10

160 knots
Specific Range

- Distance traveled per unit fuel.
- Similar to miles per gallon on automobiles.
- Generally expressed as nautical miles traveled per pound of fuel.
- If one knows the power required by the helicopter as a function of forward speed, and the fuel consumption of the engine as a function of horse power, we can compute fuel flow rate at any given forward speed.
- For multiple engines, divide power required by the number of engines to obtain power required per engine.
Graphical Determination of Maximum Specific Range for a given GW

Fuel Flow Rate lb/hr

Relative Forward Speed including head or tail wind, knots

Best SR = 1/slope

Best speed for maximum specific range
Range of the Aircraft

\[ R = \int_{start}^{end} V dt = \int_{start}^{end} V \frac{dW}{\text{fuel consumption, lb/sec}} = \int_{GW_{\text{take-off}}}^{GW_{\text{landing}}} SR d(GW) \]

Range is found by numerical integration of the curve below, Generated by computing the best SR for various gross weights.
Payload Range

• Provides an indication of the helicopter for carrying useful loads.
• This information can be extracted from the range calculations described previously.
• Plotted as Range vs. Payload, for various payload conditions, for a fixed maximum take-off weight.
• As payload increases, the amount of fuel that can be carried decreases, and the range decreases.
Payload Range

GW=28000 lb (auxiliary tanks)

Payload Range, Nautical Miles

Payload, lb

GW=28000 lb (auxiliary tanks)

20,000 lb

10,000 lb

400

1600
Endurance

• In some military missions and search and rescue operations, loiter is more important than range.
• Endurance is defined as the hours of loiter per pound of fuel.
• Loiter is done at the forward speed where power consumption is minimum.
Power Consumption at a given GW

- Power Consumption
- Forward Speed
- Power HP
- Power loiter
- Best speed for Loiter

Diagram showing the relationship between power consumption and forward speed, indicating the best speed for loiter.
Specific Endurance

- Specific endurance is defined as \( 1 / (\text{fuel consumption in lb/hr}) \) under loiter conditions.
- Once we know the power consumption at loiter speed from the previous chart, we can use engine performance data to find fuel consumption in lb/hr at this power.
- The inverse of this is SE.
Total Endurance

\[ \text{Total Endurance, hr} = \int_{\text{Start}}^{\text{End}} dt \]

\[ = \int_{G W_{\text{takeoff}}}^{G W_{\text{landing}}} \frac{dW}{SE} \]
Rate of Climb

\[ R/C, \text{ ft/min} = \frac{\text{Power Available} - \text{Power Required}}{\text{Gross Weight}} \]

Diagram:
- Power Available
- Power Required
- Maximum Forward Speed
- Forward Speed
- Power
Absolute Flight Ceiling: (Power required=Power Available)
Service Ceiling: 100ft/min climb possible
Helicopter Performance

Special Performance Problems
Special Performance Problems

• Turns and Pull-Ups
• Autorotation
• Maximum Acceleration
• Maximum Deceleration
Load Factor in Steady Turn

Load Factor $= n = \frac{\text{Thrust}}{\text{Weight}}$

During turns and maneuvers, additional thrust must be generated to overcome centrifugal forces.

$$n = \sqrt{1 + \left(\frac{V\dot{\theta}}{g}\right)^2}$$

$$\dot{\theta} = \frac{V}{R}$$

Power needed to perform a turn is analyzed in the same way as steady level flight. The gross weight is multiplied by $n$. 

Radius of Turn, $R$
Turn while losing altitude and speed

- In some cases, the engine can not supply the power needed to perform a steady level turn at a constant altitude.
- The pilot will lose vehicle velocity, and altitude.
- In other words, some of the vehicle kinetic energy and potential energy are expended to perform the turn.
- In that case, the average velocity may be used to compute the load factor $n$ and the thrust/power needed.
- From the energy required to perform the turn, subtract off the energy extracted from the change in the vehicle kinetic energy during the turn, and subtract off the energy extracted by a change in the vehicle potential energy.
- The rest is the energy that must be supplied by the engine.
- Divide the energy by time spent on the turn to get power needed.
Load Factor in Pull-Up Maneuvers

\[ T = GW + CF \]

\[ = GW + \frac{GW}{g} R \dot{\theta}^2 = GW \left[ 1 + \frac{V \dot{\theta}}{R} \right] \]

Power is computed as in Steady level flight.
Nap-of-the-Earth Flight

\[ T = GW - CF \]

\[ = GW - \frac{GW}{g} R \dot{\theta}^2 = GW \left[ 1 - \frac{V \dot{\theta}}{R} \right] \]
Autorotation

• In the case of power failure, the RPM rapidly decays. How fast, how much?
• The vehicle supplies power to the rotor by descending rapidly. How fast?
• The vehicle continues to move forward. How far?
Inertia of the Entire System

The kinetic energy stored in the system is determined by the moment of inertia of the entire system - main rotor, tail rotor, shaft, etc.

\[ J = \text{Moment of Inertia} = \]

\[ J_{\text{Main}} + J_{\text{Tail}} \frac{\Omega_{\text{tail}}}{\Omega_{\text{main}}} + J_{\text{transmission}} \]
Time Available

Time available before rotor stops completely is estimated as initial kinetic energy of the entire system divided by torque times $\Omega$ needed to keep the rotor spinning.

$$t_{KE} = \frac{1}{2} J \Omega^2_0$$

$$\frac{2}{\Omega_0 Q}$$

A large enough moment of inertia is needed to buy the pilot time to enter into autorotative mode. The torque must be as small as possible. Pilot will reduce collective pitch as needed to reduce torque.
Variation in Angular Velocity following Power Failure

We assume that torque coefficient roughly remains the same.

Torque is proportional to $\Omega^2$

Rate of change of angular momentum of the entire system equals torque acting on the system.

$$J \frac{d\Omega}{dt} = -Q_0 \left( \frac{\Omega}{\Omega_0} \right)^2$$

Integrate: $$\frac{\Omega}{\Omega_0} = \frac{1}{1 + \frac{Q_0 t}{J\Omega_0}}$$

A high $J$ and high $\Omega_0$ needed to keep rotor spinning at a high enough speed.
Rotor Speed Decay following Power Failure

\[ \frac{\Omega}{\Omega_0} \]

- \( t_{KE} = 4 \text{ sec} \)
- \( t_{KE} = 1 \text{ sec} \)

Time: 3 to 4 sec
Descent Angle

\[ \text{Descent Angle} \approx \frac{V_{\text{Descent}}}{V_{\text{Forward}}} \]
Once the autorotative mode is entered...

Power is supplied by the air, as in a wind mill. The RPM stabilizes.

Rate of Descent = \( \frac{\text{Power required}}{\text{Gross Weight}} \)

We can plot rate of descent as a function of forward speed.
Rate of Descent in Autorotation

Rate of Descent = \( \frac{P}{GW} \)

Best Forward Speed
For minimum descent rate

Best descent angle

Too steep a descent

Vortex ring state
Encountered..

Speed for minimum Descent angle feasible
Zoom Maneuver

Aircraft has high forward speed when power failure occurs. Vehicle is too low.

Pilot zooms to higher altitude, trading kinetic energy for potential energy.

Autorotative descent is attempted.

Ground
Deadman’s Curve
See NASA TND-4336

Avoid!
Power consumption is too high.
Vortex ring state possible

Avoid. High speed impact with ground likely
Minimum Touchdown Speed

Collective Flare
Increase collective.
Bring nose down.
Touch down as slowly as possible

Cyclic Flare:
Cyclic pitch is increased to
Increase lift, tilt rotor aft, slow down
descent rate.
Vehicle pitches up.

Steady Autorotative Descent
Thumb Rule: For safe autorotative landing employing flare, the autorotative index must be higher than 60 for single engine helicopters, and higher than 25 for twin-engine aircraft (assuming only one engine is likely to fail).
Maximum Acceleration in Level Flight

- We first find the highest fuselage equivalent flat plate area, $f_{\text{max}}$, at which steady level flight is possible.
- This is done by assuming various values of $f$, and computing power needed in forward flight using methods described earlier.
- $f$ is increased to its maximum value $f_{\text{max}}$ until power needed = power available.
- Compute maximum thrust = maximum drag = $\frac{1}{2} \rho f_{\text{max}} V^2$
- Compute actual thrust needed = $\frac{1}{2} \rho f_{\text{max}} V^2$
- The maximum acceleration = $(\text{Maximum thrust} - \text{Actual Thrust})/m$, where $m$ is the mass of the aircraft.
Maximum Deceleration

- When the vehicle decelerates, the pilot tilts the rotor disk aft, so that thrust is pointing backwards, and vehicle slows down.
- If deceleration occurs too quickly, autorotation may occur, and the rotor RPM may increase too much, and structural limits may be exceeded.
- To avoid this, only a 10% to 20% overspeeding of the blade RPM is permitted.
- Compute the H force, tip path plane angle, T, fuselage drag fq, etc. at this higher permitted RPM, for autorotative conditions.
- Compute the maximum permissible rearward directed force $= -T \alpha_{TPP} + H + f_q$
- Maximum deceleration is this force divided by helicopter mass.