Prescribed Wake Models for Rotors in Forward Flight

1. Rigid Wake:

In the rigid wake model, the tip vortex position is described by its age $\phi$, which is the current blade azimuthal angle position minus the azimuthal angle of the shedding initiation point. The wake geometry is described in the inertial coordinate system as:

$$
\begin{align*}
    x &= r \cos(\psi - \phi) + \mu_x \phi + \lambda_x \phi \\
    y &= r \sin(\psi - \phi) + \mu_y \phi + \lambda_y \phi \\
    z &= z_0 + \mu_z \phi - \lambda_z \phi
\end{align*}
$$

(1)

where $\psi$ is the azimuthal angle of the reference blade, and $\phi$ is the vortex wake age.

The Glauert uniform inflow model is used to estimate the inflow components $\lambda_x$, $\lambda_y$ and $\lambda_z$. The induced velocity in dimensionless form is:

$$
\dot{\lambda} = \mu \tan \alpha + \lambda_z = \mu \tan \alpha + \frac{c_r}{2 \sqrt{\mu^2 + \lambda^2}}
$$

(2)

Landgrebe’s Prescribed Wake Model:

In this model, the x and y coordinates of the tip vortex are prescribed from a rigid wake model, as shown above. The vertical displacements of the tip vortices are given as:

$$
\frac{z_V}{R} = -\lambda_z \phi - EG
$$

(3)

where $E$ is an envelop function given by:

$$
E = A_0 \phi \exp(A_1 \phi) \quad \text{if } \phi \leq 4\pi \\
E = M\phi + B \quad \text{if } \phi \geq 4\pi
$$

(4)

and

$$
G = \sum_{n=0}^{N} C_n \cos n\phi + D_n \sin n\phi
$$

(5)

Beddoes Prescribed wake Model:

In this model, the x and y coordinates of the tip vortex are prescribed from a rigid wake model, as shown above. The wake skew angle $\chi$ is next determined. Next, a new constant $E = \chi/2$ is computed. Beddoes assumes that the vertical velocity at which a vortex filament descends is given by

$$\lambda_i = \lambda_0 \left[ 1 + E \frac{x}{R} - E \frac{y}{R}^{3/2} \right]$$

if the filament is underneath the rotor disk. If the filament is downstream of the rotor disk, then it is assumed to move downwards at a velocity given by

$$\lambda_i = 2\lambda_0 \left[ 1 - E \frac{y}{R}^{3/2} \right]$$

The vertical position of the vortex filament is thus given by:

$$\frac{z_i}{R} = -\mu_z \phi + \int_0^\phi \lambda d\phi$$

The above integral may be numerically evaluated. The integrand will depend on whether the filament is underneath the rotor disk, or downstream of the rotor disk. Ref: Beddoes, T. S., “A Wake Model for High Resolution Airloads,” Second International Conference on Basic Rotorcraft Research, 1985, Research triangle Park, North Carolina.